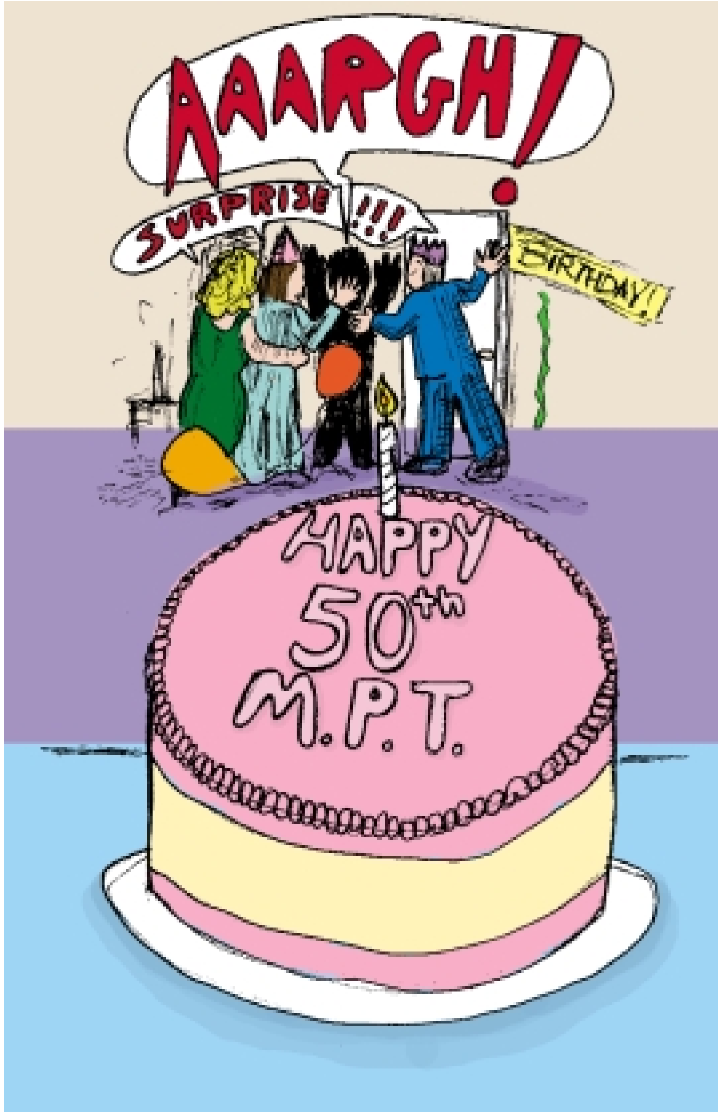
Aaron Brown

# Modern Portfolio Theory at Fifty

It's a good idea to examine foundations. Do we still need the theory or does it have only historical interest today? If we still need it, can we gain insight by reconsidering it in light of fifty years of progress?

A half century ago, as part of the post-war applied mathematics revolution, Harry Markowitz[1] came up with Modern Portfolio Theory. The brilliant concept that came to Markowitz one afternoon in the University of Chicago library (while reading John Burr Williams's *Theory of Investment Value*) is the foundation of quantitative finance.

Even among finance professionals, MPT is often confused with the Efficient Market Hypothesis and the Capital Asset Pricing Model.[2] So let's start with the definitions. MPT is a theory, "a set of statements or principles devised to explain a group of facts or phenomena, especially one that has been repeatedly tested or is widely accepted and can be used to make predictions about natural phenomena."[3] It holds that investors care about the statistical properties of their portfolios.




*Something suggested that MPT had stopped the HRT on the QT*

EMH is a *hypothesis*, "something taken to be true for the purpose of argument or investigation; an assumption." It states that securities are priced as if markets incorporate all information. CAPM is a *model*, "a schematic description of a system, theory, or phenomenon that accounts for its known or inferred properties and may be used for further study of its characteristics." It says that the excess expected return on any capital asset is equal to its Beta times the excess expected return on the market portfolio.

There is no point in arguing about whether theories, hypotheses or models are true. Theories are useful if they stimulate a useful body of work. Hypotheses are useful if they generate interesting conclusions that facilitate learning. Models are useful if their predictions are accurate enough for practical purposes.

## Modern Portfolio Theory

The hard thing about teaching MPT today is explaining to students why it is not obvious. That's one measure of its success. But a little thought reveals that most things people buy are not evaluated by statistics. If you can measure or estimate the value of something accurately enough, the statistical properties are not very important. Even if there is a large degree of uncertainty, if you are close to risk neutral, all you care about is the expected value. So we only consider statistical measures when the uncertainty is very large compared to the value, and the purchase is important enough that non-linear utility is significant.

Even in that case, statistics may not be helpful. There is certainly a large degree of uncertainty involved when choosing a career or a spouse, for example, but there aren't a lot of useful statistics to help with the problem.

So when Markowitz asserted investors care about statistical properties, he implied that research and analysis could not produce security values accurate enough for decision-making. He also implied that there was enough high-quality data for useful statistical analysis.

These things were just starting to become true in the early 1950s. Before the market reforms of the 1930s, it was not hard to uncover useful non-public information, mainly because there was so little public information. Investors wanted inside information to bet on sure things, not statistics about random investments. Statistical investing before the end of World War Two would have been a quick road to ruin. Moreover the volatility of the market was dominated by panics, wars, expropriations and other crises that are hard to model, especially without high quality time series data and computers.

Another obstacle to acceptance of MPT was the idea that statistics is the same as gambling. Stock markets, and commodity markets even more, struggled against legal discouragement and moral disapprobation of gambling. Professionals insisted markets were games of skill, not chance, which today would bring the Justice Department running with criminal indictments.

The other half of MPT is that investors analyze an entire portfolio. In other words, the value of a security depends not on its stand-alone statistical characteristics, but on how it can contribute to a portfolio. That's natural enough in theory, but to be important in practice it

**Another obstacle to acceptance of MPT was the idea that statistics is the same as gambling. Stock markets, and commodity markets even more, struggled against legal discouragement and moral disapprobation of gambling**

requires a lot of difference in covariance structure among securities, relative to differences in expected return. If all securities were independent, for example, then it would make sense to select the highest expected return ones. You would not need any sophisticated portfolio calculation. If you don't know much about covariances, because they are hard to estimate given the available data and computing power, it also makes sense to select securities one at a time. If expected return differences are very great, you can afford to decide on that basis alone. But if expected returns are close and covariances differ measurably, you need to consider portfolios as a whole.

In general, we like to separate decisions. It's easier to make them that way. Some people buy a shirt if they like it. Other people plan wardrobes and think about whether the shirt goes with other clothing and fills a gap. Some people eat what they feel like at the time; others plan diets to balance calories and nutrition. Some sports teams draft the best available athlete; others believe in taking players to fill specific needs. MPT says securities should be analyzed like wardrobes/diets/specific needs. However, it also says that we need not consider broader interactions. It does not tell us to select our careers, houses, spouses and breakfasts along with our securities to maximize total utility.

MPT does not say that markets are efficient. It's possible for each investor to have her own views on statistical properties, and therefore select portfolios that appear inefficient to other investors. Therefore it can never be proven or disproven, except in the irrelevant sense as a statement about investor psychology (which it clearly does not describe, investors mostly care about how much money they made or lost, not abstract statistical properties).

## Efficient Market Hypothesis and the Capital Asset Pricing Model

EMH was invented by Eugene Fama[4] in the 1960s to permit scientific investigation of financial theories. Without it, you can explain any price of any security as a disagreement among investors. No statement can be disproved, so no statement can be proven. But if you assume all investors agree about the statistical properties of securities, you can make strong statements about the relation of prices to observed price movements. EMH led to an explosion of empirical work that stunned everyone, including EMH proponents, by how efficient markets actually were. What started as a convenient simplification for approximate work, like ignoring air resistance in high school physics problems, ended up as a plausible exact description of the world.

Of course a few of these studies turned up anomalies. Most of the anomalies disappeared with theoretical advances and better data. Some remain today. However, no anomaly was ever discovered by anyone who didn't start out assuming EMH. And no one has ever come up with an alternative to EMH that generates testable predictions with better accuracy.

William Sharpe[5] is the person most responsible for developing the CAPM. This model builds on both MPT and EMH. If investors care about the statistical properties of their portfolios (MPT), they should hold "efficient" portfolios,[6] those that maximize expected return for a given level of risk. If all investors agree on the statistical properties of securities (EMH) then with a few extra assumptions, the combination of any two efficient portfolios is efficient. That means the market portfolio of all securities, which is a combination of all individual investor securities, must be efficient.

Consider adding or subtracting a small amount of security $S$ to the market portfolio. The expected return will go to:

$$(1 - w)R_m + wR_S$$

where $w$ is the amount you add, $R_m$ is the expected return on the market portfolio and $R_s$ is the expected return on security $S$. The variance will go to:

$$(1 - w)^2 V_m + w^2 V_S + \rho_{S,m} w(1 - w)\sqrt{V_m V_S}$$

where $V_m$ is the variance of the market portfolio, $V_S$ is the variance of the security and $\rho_{S,m}$ is the correlation coefficient between the security and the market portfolio. Even if investors use a measure of risk other than variance, it is likely that an analog of this equation is close to true for a redefined $\rho$.

The ratio of the derivative with respect to $w$ of the change in expected return to the derivative of risk should be the same for security $S$ and the risk-free asset, evaluated at $w = 0$. Otherwise you could get a better expected return than the market portfolio with the same level of risk by changing the proportion of security $S$ and borrowing or lending at the risk-free rate. So we have the equation:

$$\frac{R_f - R_m}{V_m} = \frac{R_S - R_m}{V_m - \rho_{S,m}\sqrt{V_m V_S}}$$

because for the risk-free investment $V_S$ is zero.

A little algebra gives:

$$\rho_{S,m}\sqrt{\frac{V_S}{V_m}} = \frac{R_S - R_f}{R_m - R_f}$$

or

$$(R_S - R_f) = \beta_{S,m}(R_m - R_f).$$

I like to reproduce that because people forget how simple the math is. A different definition of risk, if it is not too exotic, will give a similar formula with a somewhat different definition of Beta. And no reasonable utility function can get too far from this result.

The more problematic assumption is that the market portfolio is efficient. This relies on some unrealistic assumptions. However, even if it is false, it seems unlikely that there are a large number of distinct efficient portfolios that cannot aggregate. So we might have a model with three or four Betas corresponding to different portfolios (that collectively add up to the market), but it would be surprising to have dozens.

## Where are they now?

It is easy to look around and find things inconsistent with MPT. How could any investor concerned with the statistical properties of her portfolio pay 43.26 per cent of assets every year to invest in a mutual fund that performed in the bottom 1 per cent of all fund in 9 of the last 10 years? Or pay 2.35 per cent per year to invest in a municipal bond mutual fund? Equity research reports devote a lot more space to justifying non-stochastic valuations of securities than describing the statistical properties. Executives go to jail for misrepresenting assets and liabilities, never for fraudulent statistical properties. Legal "prudent man" investing rules require arbitrary figures, but no knowledge of statistics. In personal finance, you spend a lot more time worrying about taxes, fees, investment selection and retail services than detailed statistical analysis of the portfolio.

But this is just the froth. Statistical analysis is deeply entrenched in all professional finance. Mutual funds analysis is almost entirely statistical. Statistical models have made great inroads in credit. Credit rating agencies still avoid specific probability statements, but since 1980 they have provided extensive statistical information about rating migrations, defaults and recoveries. Financial regulation and tax law have increasingly taken a statistical view. Looking at the big picture, MPT has clearly won the day. It is still wiping out pockets of resistance, with no signs of flagging energy. Increasingly, finance is statistics.

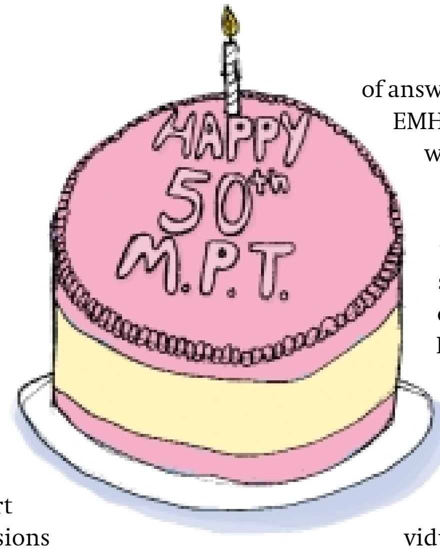
However, there is one major challenge to MPT. Once Fischer Black and Myron Scholes published their famous option pricing model[7] it became possible to observe volatility directly. Rapid expansion of derivatives trading means we now measure market implied statistical properties much better than actual ones. The two do not correspond well.

Two obvious explanations for this difference argue against MPT. One likely explanation is the statistical properties are more complicated than current models allow. But how can investors care about things that we cannot even model, much less measure? Another explanation is there are systematic, non-statistical factors that affect security price movements. But MPT cannot explain these. A third explanation, that markets are highly inefficient, is consistent with MPT, but hard to accept for other reasons.

MPT has generated this contradiction itself. The thesis energized a charge that broke through barriers to make markets more efficient. The more efficient they became, the more the thesis became true, but the more rigorous the tests became. This generated the antithesis of implied statistical properties. Unless someone can reconcile implied and actual statistical prop-

**MPT has clearly won the day. It is still wiping out pockets of resistance, with no signs of flagging energy. Increasingly, finance is statistics**

## The more things become statistical, the greater the pressure to reconcile actual and implied statistical properties




erties as the same, we will need a new thesis, a synthesis.

EMH is unchallenged as a basis for research, but faces a lot of opposition in practice. There appear to be systematic mispricings that exist over long periods of time and are hard to reconcile with market efficiency. The spread between long-term interest rates and short-term, the spread between equity returns and low-risk investments, the spreads between interest rates in different currencies and the currency price movements, the spread between mortgage securities and bonds; all defy reasonable efficient explanation. Within asset categories, some quantitative hedge funds have compiled records that seem to beat the category average consistently on a risk-adjusted basis.

However, all these anomalies are small, either a small number of basis points or exploited only by small investment funds. If we define an inefficiency as amount of excess return per year times amount of money exploiting it, there may be some $10 million anomalies, but no $100 million ones. While these amounts can be attractive individually, overall they represent only a few basis points on the global financial system. Moreover, it's always possible that they will be explained by a refined theory.

The CAPM naturally suffers from the problems listed for MPT and EMH. If you try to fit it with implied market statistics, it is clearly false. The EMH anomalies contradict it as well. However, it remains a very useful model. Expected return is so hard to measure that you cannot convincingly demonstrate that the equity premium for all securities over all history is even positive. So we do not have a hope of measuring expected return for a specific asset over any investment horizon of interest to humans. Beta and the risk-free rate we can measure reasonably accurately with short time series. For decisions that depend only on relative excess return, we do not need to know the expected return on the market portfolio. If we need to make decisions based on absolute expected return, we have a prayer of success using trying to guesstimate the expected return on the market, no chance at all of doing it for individual securities.

So even if the CAPM is misspecified, the errors from that are negligible compared to the other errors in estimating expected return. It's a matter of opinion whether you get better results assuming a single efficient market portfolio or using multiple Betas. For estimating Beta, what data and estimation technique you use is much more important that what theoretical definition you choose.

## The future

I'm not bold enough to predict the next fifty years, but for the foreseeable future I expect MPT to continue vanquishing non-statistical interference with markets. The more things become statistical, the greater the pressure to reconcile actual and implied statistical properties. This will require a theoretical breakthrough, which could replace MPT, or strengthen it.

As long as MPT survives, I cannot see any alternative to EMH for theoretical work. There are plenty of anomalies to work with, but they do not seem to point in consistent directions. People will continue to get partial answers from other approaches, such as behavioral finance or chaos theory, but none of these holds out much hope of answering the central questions of finance. So EMH will probably be with us for a while as well.

The thing that make the CAPM so hard to prove (or disprove) are the same things that make it useful. Even if it's false, it should be so close to the truth that we cannot measure the difference. While the EMH will survive for theory, the CAPM will survive for practice. However, I think CAPM will be replaced before MPT or EMH. As data improves, both because we have longer time series, more securities and higher quality individual data, we should be able to refine measurement enough to discover a more precise model.

I predict that all of us can use MPT and EMH until retirement, but I don't recommend having CAPM tattooed on your arm.

## REFERENCES

1. Markowitz, H. M. Portfolio selection, *Journal of Finance* 7 (March 1952), 77-91. Of course, Markowitz built on ideas of others, and many other people contributed to the development of the theory. This is also true about Fama and EMH, and Sharpe and CAPM.
2. Another common term, Random Walk Theory, was originally synonymous with MPT, but later came to mean something closer to EMH. 'Random walk' can mean 'stochastic process.' It is sometimes also used to mean either 'Markov process' or 'martingale.' Modern Portfolio Theory uses the term in the first sense, the other senses imply a degree of market efficiency.
3. All definitions form the *American Heritage Dictionary of the English Language*, Fourth Edition
4. First clearly stated in a journal in Fama, Eugene F., Random walks in stock market prices, *Financial Analysts Journal*, September/October 1965
5. Sharpe, William F., A simplified model for portfolio analysis, *Management Science*, Vol. 9, 1963
6. 'Efficient' here means something entirely different than in 'efficient' market hypothesis, and there is a another sense of 'efficient' markets, meaning no transaction costs. This has confused generations of finance students.
7. Black, Fischer and Scholes, Myron,. The pricing of options and corporate liabilities, *Journal of Political Economy*, 81:637–659, 1973

W